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# Energy level statistics in weakly disordered systems: from quantum to diffusive regime

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## Abstract

We calculate two-point energy level correlation function in weakly disordered metallic grain by taking account of localization corrections to the universal random matrix result. Using supersymmetric nonlinear  $\sigma$  model and exactly integrating spatially homogeneous modes, we derive the expression valid for arbitrary energy differences from quantum to diffusive regime for the system with broken time reversal symmetry. Our result coincides with that obtained by Andreev and Altshuler (1995 *Phys. Rev. Lett.* **72** 902) where homogeneous modes are perturbatively treated.

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## 1. Introduction

Much research interest has been attracted by the universal properties of quantum systems with randomness; the randomness arises from various sources such as complexity of the system for the case of, e.g., complex nuclei, stochasticity for disordered conductors and instability of classical trajectories for chaotic systems [1]. If wavefunctions have enough time to diffuse throughout the system, the spatial structure of the system becomes immaterial. For such cases, the system is described as belonging to the ergodic regime where the statistical properties have been extensively studied. For example, correlation functions of the energy levels [2] or the scattering matrices [3, 4] are shown to have the universal forms where specific features of individual systems are represented by a few parameters [1]. These universal forms can be calculated using standard random matrix models belonging to pertinent Gaussian ensembles.

If the relevant time scale is comparable with the diffusion time, we need to turn to spatial dependent parts of the correlation functions which are not included in the universal random matrix results. These spatial dependent parts for weakly disordered systems are the subject of this paper. The weakly disordered systems are characterized by the condition that  $g \gg 1$  where  $g$  is the dimensionless conductance. The spatial dependent parts of the correlation functions are expected to appear as higher order corrections with respect to  $1/g$  to the universal random matrix results. The two-point energy level correlation function  $R(s)$  (see equation (1.1))

with corrections up to  $1/g^2$  has been calculated by several authors: in their seminal work, using diagrammatic perturbation theory, Altshuler and Shklovskii [5] obtained the smooth part of  $R(s)$  valid for large energy differences  $s \gg 1$ . Deviations from the universal statistics are determined by the diffusive classical dynamics of the system. However, diagrammatic perturbation does not give an oscillatory part of  $R(s)$  characteristic of the universal random matrix result.

The  $1/g^2$  corrections including the oscillatory part were first obtained by Kravtsov and Mirlin (KM) [6] using the supersymmetric  $\sigma$  model [7], the effective theory of a particle moving through a disordered conductor. Their result is, however, valid only for small energy differences  $s \ll g$  [8]. Later, also using the supersymmetric  $\sigma$  model, Andreev and Altshuler (AA) [9] claimed that the oscillatory part can be obtained from perturbation around a novel saddle point in addition to the ordinary one. Their result relies on perturbation and hence is restricted to the region of large energy differences  $s \gg 1$ . The AA result was subsequently reproduced using the replica [10] and Keldysh [11]  $\sigma$  models. In quantum chaotic systems, the analyses based on the semiclassical periodic orbit theory [12] and the ballistic  $\sigma$  model [13] led to results similar to those of AA.

In all these results, deviations from the universal form are expressed in terms of eigenvalues of a certain operator or its spectral determinant. This operator is a diffusion operator for the case of disordered conductors or a Perron–Frobenius operator for quantum chaotic systems [14]. We can surmise from these results that universal forms also exist in the deviations from universal random matrix results where parameters representing system specific features are now the spectra of this operator.

At present, the available results are yet insufficient because KM’s result is valid for  $s \ll g$  and AA’s one is valid for  $s \gg 1$ . The method to calculate an expression valid for the entire range of energy differences is desirable<sup>1</sup>. This paper presents one such method. Extending the KM procedure, we calculate  $1/g^2$  corrections to the two-point energy level correlation function

$$R(s) = \Delta^2 \langle \rho(E) \rho(E + \omega) \rangle \quad (1.1)$$

valid for arbitrary  $\omega$ , where  $s = \omega/\Delta$ ,  $\Delta$  is the mean level spacing,  $\rho(E)$  is the density of states at energy  $E$ , and  $\langle \dots \rangle$  denotes averaging over realization of the random potential. We consider disordered systems with broken time reversal symmetry because the pertinent random matrix ensemble (unitary ensemble) is the simplest among the three classical ensembles (unitary, orthogonal and symplectic). We follow the KM procedure but do not use the  $\omega$  expansion which limits the range of validity of their results to small  $\omega$ . As a result, we verify that the AA expression gives the correct  $g \gg 1$  asymptotic at arbitrary  $\omega$  for unitary ensemble. For the unitary case, invoking a mathematical theorem, it is implicitly shown [15] that AA’s result is exact in spite of their use of perturbation. Here, we want to show it by explicit calculation which we expect to be applicable in other ensembles with some modifications.

## 2. Model and method

The supersymmetric  $\sigma$  model is a field theory whose variables  $Q$  are supermatrices [7]. Statistical properties of a particle moving through random potentials can be derived from a generating functional of  $Q$ . The expression for  $R(s)$  reads

$$R(s) = \frac{1}{16V^2} \text{Re} \int dQ e^{-S(Q)} \left[ \int d\vec{r} \text{str} k \Lambda Q(\vec{r}) \right]^2 \quad (2.1)$$

<sup>1</sup> The supersymmetric  $\sigma$  model description is valid under the condition  $\omega \ll 1/\tau$  where  $\tau$  is the elastic mean free time (see [7]). The energy difference  $\omega$  considered in the paper is assumed to satisfy this inequality.

$$S(Q) = \frac{\pi}{4V} \int d\vec{r} \operatorname{str} \left[ \frac{D}{\Delta} (\nabla Q(\vec{r}))^2 + 2is^+ \Lambda Q(\vec{r}) \right]. \quad (2.2)$$

Here  $\operatorname{str}$  denotes the supertrace,  $Q(\vec{r}) = T(\vec{r})^{-1} \Lambda T(\vec{r})$  is a  $4 \times 4$  supermatrix with  $T(\vec{r})$  belonging to the coset space  $U(1, 1|2)$ ,  $\Lambda = \operatorname{diag}(1, 1, -1, -1)$ ,  $k = \operatorname{diag}(1, -1, 1, -1)$ ,  $V$  is the system volume,  $D$  is the classical diffusion constant,  $s^+ = s + i0^+$ , and  $0^+$  is positive infinitesimal. We use the notation of [7] everywhere.

The ordinary saddle point is  $Q = \Lambda$  and AA's novel one is  $Q = -k\Lambda$ . The expansion around these saddle points and calculation of the functional integral by Gaussian approximation give AA's result. The quadratic term of the action  $S(Q)$  defines the diffusion propagator in terms of which a perturbation series is constructed by Wick's contraction. When  $s$  goes to zero, the diffusion propagator diverges for spatially homogeneous modes (which we hereafter call the zero-momentum modes, or zero modes for short, because their momentum is zero). Thus their approach is restricted to the region  $s \gg 1$ .

KM have overcome the difficulty of zero mode divergence by treating the zero modes and the other spatially inhomogeneous modes with non-zero momenta (hereafter called non-zero modes) separately. The matrix  $Q(\vec{r})$  is decomposed in the following way [6],

$$Q(\vec{r}) = T_0^{-1} \tilde{Q}(\vec{r}) T_0 \quad (2.3)$$

where  $T_0$  is a spatially homogeneous matrix from the coset space and  $\tilde{Q}(\vec{r})$  describes all non-zero modes (see equations (2.4) and (2.5)). After perturbative calculation of the contributions by non-zero modes, there remains a definite integral over a few zero mode variables, which can be evaluated without using perturbation. This amalgamation of both perturbative and non-perturbative calculation, in principle, could give the weak localization corrections valid for entire range of  $s$ . In the course of calculation, however, assuming small  $\omega$ , KM expand the exponential of the energy term (the second term of the action (2.2)). Thus their result is restricted to  $s \ll g$ .

The difficulty arising when the energy term is kept on the shoulder of exponent is that the propagators of non-zero modes become dependent on zero mode variables. Then after eliminating non-zero modes by perturbation, the resulting zero mode integral contains infinite sums and products of non-zero mode propagators (see equation (5.5)) as an integrand. Hence the zero mode integral, though still being definite over a few variables, does not seem feasible. In this paper, we show that this is not the case: by replacing the order of integration, it turns out that we can first integrate the zero mode exactly. Then integration over the non-zero modes by perturbation can be carried out without any trouble.

We give a summary of notation: when  $\Delta \ll E_c$  (where  $E_c = D/L^2$  and  $L$  is the system size), the matrix  $\tilde{Q}(\vec{r})$  fluctuates only weakly around  $\Lambda$ . Then the fluctuations can be treated by perturbation with the expansion parameter  $1/g$  (where  $g = E_c/\Delta$ ). A convenient parametrization is

$$\tilde{Q}(\vec{r}) = (1 - W(\vec{r})/2) \Lambda (1 - W(\vec{r})/2)^{-1} \quad (2.4)$$

where

$$W(\vec{r}) = \begin{pmatrix} 0 & W_{12}(\vec{r}) \\ W_{21}(\vec{r}) & 0 \end{pmatrix} \quad (2.5)$$

$W_{21}(\vec{r}) = k W_{12}(\vec{r})^\dagger$ .  $W_{12}(\vec{r})$  is expanded as  $W_{12}(\vec{r}) = \sum_{\vec{q} \neq 0} \phi_{\vec{q}}(\vec{r}) W_{12\vec{q}}$  where  $\phi_{\vec{q}}$  is an eigenfunction of the diffusion operator  $-D\nabla^2$  with an eigenvalue  $D|\vec{q}|^2$ .  $\{\phi_{\vec{q}}(\vec{r})\}$  constitutes a complete orthonormal set and  $\phi_0(\vec{r}) = 1/\sqrt{V}$ . Thus  $\int d\vec{r} W(\vec{r}) \propto \int d\vec{r} W(\vec{r}) \phi_0(\vec{r}) = 0$ . The Jacobian of the transformation expressions (2.3) and (2.4) from the variable  $Q$  to  $\{T_0, W\}$  is

$$J(W) = 1 - \frac{1}{16V^2} \int d\vec{r} d\vec{r}' (\text{str } W_{12}(\vec{r}) W_{21}(\vec{r}'))^2 + O(W^5). \quad (2.6)$$

The derivation is given in appendix A.

The spatially homogeneous supermatrix  $T_0$  is parametrized in a quasi-diagonalized form

$$T_0 = U^{-1} \hat{T}_0 U \quad (2.7)$$

with ‘eigenvalue’ matrix  $\hat{T}_0$  given by

$$\hat{T}_0 = \begin{pmatrix} \cos \frac{\hat{\theta}}{2} & i \sin \frac{\hat{\theta}}{2} e^{i\hat{\varphi}} \\ i \sin \frac{\hat{\theta}}{2} e^{-i\hat{\varphi}} & \cos \frac{\hat{\theta}}{2} \end{pmatrix} \quad (2.8)$$

$$\hat{\theta} = \begin{pmatrix} \theta_F & 0 \\ 0 & i\theta_B \end{pmatrix} \quad \hat{\varphi} = \begin{pmatrix} \varphi_F & 0 \\ 0 & \varphi_B \end{pmatrix} \quad (2.9)$$

where  $0 \leq \theta_B < \infty$ ,  $0 \leq \theta_F \leq \pi$ ,  $0 \leq \varphi_B \leq 2\pi$ ,  $0 \leq \varphi_F \leq 2\pi$ . The ‘diagonalizing’ matrix  $U$  is given by

$$U = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \quad (2.10)$$

$$v_1 = \exp \begin{pmatrix} 0 & \xi_1 \\ -\xi_1^* & 0 \end{pmatrix} \quad v_2 = \exp i \begin{pmatrix} 0 & \xi_2 \\ -\xi_2^* & 0 \end{pmatrix} \quad (2.11)$$

where  $\xi_1$  and  $\xi_2$  are anticommuting variables. For this parametrization, the measure  $dT_0$  is given by

$$dT_0 = \frac{\sinh \theta_B \sin \theta_F}{(\cosh \theta_B - \cos \theta_F)^2} d\theta_B d\theta_F d\varphi_B d\varphi_F d\xi_1 d\xi_1^* d\xi_2 d\xi_2^*. \quad (2.12)$$

### 3. Changing the variables

To simplify the zero mode integration, we change the variables. Substituting the decomposition (2.3) into equation (2.1), we obtain

$$\text{str}(\nabla Q(\vec{r}))^2 = \text{str}(\nabla \tilde{Q}(\vec{r}))^2 \quad (3.1)$$

$$\int d\vec{r} \text{str} \Lambda Q(\vec{r}) = \text{str} \Lambda T_0^{-1} Y T_0 \quad (3.2)$$

$$\int d\vec{r} \text{str} k \Lambda Q(\vec{r}) = \text{str} k \Lambda T_0^{-1} Y T_0 \quad (3.3)$$

where  $Y = \int d\vec{r} \tilde{Q}(\vec{r})$ . We make the supermatrix  $Y$  a block diagonal form as follows (for details see appendix B),

$$e^X Y e^{-X} = \hat{Q} \quad (3.4)$$

where  $X$  is a block off-diagonal supermatrix and

$$\hat{Q} = \Lambda \left( V + \frac{1}{2} \int d\vec{r} W^2(\vec{r}) + \frac{1}{8} \int d\vec{r} W^4(\vec{r}) + O(W^6) \right). \quad (3.5)$$

Up to  $1/g^2$  order calculated here,  $e^X T_0$  can be replaced with  $T_0$  (see appendix B). Thus in equations (3.2) and (3.3),  $Y$  can be replaced with  $\hat{Q}$ .

To eliminate the anticommuting variables of the zero modes in the action  $S(Q)$ , we change the variables as  $U W U^{-1} \rightarrow W$ . Eventually, we obtain

$$\int d\vec{r} \operatorname{str} \Lambda Q(\vec{r}) = \operatorname{str} \Lambda \hat{T}_0^{-1} \hat{Q} \hat{T}_0 \quad (3.6)$$

$$\int d\vec{r} \operatorname{str} k \Lambda Q(\vec{r}) = \operatorname{str} k \Lambda U^{-1} \hat{T}_0^{-1} \hat{Q} \hat{T}_0 U. \quad (3.7)$$

#### 4. Integration over the zero-momentum mode variables

Expanding the pre-exponential term of equation (2.1) in the anticommuting variables of the zero modes, we obtain

$$(\operatorname{str} k \Lambda U^{-1} \hat{T}_0^{-1} \hat{Q} \hat{T}_0 U)^2 = (\operatorname{str} k \Lambda \hat{T}_0^{-1} \hat{Q} \hat{T}_0)^2 - 2\xi_1^* \xi_1 \xi_2^* \xi_2 (\operatorname{str} \Lambda \hat{T}_0^{-1} \hat{Q} \hat{T}_0)^2 + \dots \quad (4.1)$$

where we have used that  $\operatorname{str} \hat{Q} = 0$  and the dots indicate terms which vanish after integration of the zero mode anticommuting variables. We write the contribution by the first (second) term of equation (4.1) to  $R(s)$  as  $R_1(s)$  ( $R_2(s)$ ). Using the Parisi–Sourlas–Efetov–Wegner theorem [16], integration of zero mode variables in  $R_1(s)$  becomes the value of its integrand at  $\theta_B = \theta_F = 0$ ,

$$R_1(s) = \frac{1}{16V^2} \operatorname{Re} \int dW J(W) e^{-S(W)} (\operatorname{str} k \Lambda \hat{Q})^2 + O(1/g^3) \quad (4.2)$$

where

$$S(W) = \frac{\pi}{4V} \operatorname{str} \left[ \frac{D}{\Delta} \int d\vec{r} (\nabla \tilde{Q}(\vec{r}))^2 + 2is^+ \Lambda \hat{Q} \right]. \quad (4.3)$$

Integrating the zero mode anticommuting variables, we obtain

$$R_2(s) = -\frac{1}{8V^2} \operatorname{Re} \int dW d\theta_B d\theta_F J(W) \frac{\sinh \theta_B \sin \theta_F}{(\cosh \theta_B - \cos \theta_F)^2} \times e^{-S(\hat{T}_0, W)} (\operatorname{str} \Lambda \hat{T}_0^{-1} \hat{Q} \hat{T}_0)^2 + O(1/g^3) \quad (4.4)$$

$$S(\hat{T}_0, W) = \frac{\pi}{4V} \operatorname{str} \left[ \frac{D}{\Delta} \int d\vec{r} (\nabla \tilde{Q}(\vec{r}))^2 + 2is^+ \Lambda \hat{T}_0^{-1} \hat{Q} \hat{T}_0 \right]. \quad (4.5)$$

Here,

$$\operatorname{str} \Lambda \hat{T}_0^{-1} \hat{Q} \hat{T}_0 = \cosh \theta_B \operatorname{str} P(B) \Lambda \hat{Q} + \cos \theta_F \operatorname{str} P(F) \Lambda \hat{Q} \quad (4.6)$$

where  $P(B) = (\mathbf{1} - k)/2$  and  $P(F) = (\mathbf{1} + k)/2$  and  $\mathbf{1}$  is a unit matrix. Using  $\cosh \theta_B = \lambda_B$ ,  $\cos \theta_F = \lambda_F$ ,  $\operatorname{str} P(B) \Lambda \hat{Q} = -2V(1 + A)$  and  $\operatorname{str} P(F) \Lambda \hat{Q} = 2V(1 + B)$ , we obtain

$$R_2(s) = \frac{1}{2} \operatorname{Re} \int dW J(W) \exp \left[ -\frac{\pi D}{4V\Delta} \int d\vec{r} \operatorname{str} (\nabla \tilde{Q}(\vec{r}))^2 \right] I(s, W) + O(1/g^3) \quad (4.7)$$

where

$$I(s, W) = \int_1^\infty d\lambda_B \int_{-1}^1 d\lambda_F f(\lambda_B, \lambda_F) g(\lambda_B, \lambda_F) \quad (4.8)$$

$$f(\lambda_B, \lambda_F) = \frac{1}{(\lambda_B - \lambda_F)^2} e^{i\pi s^+ (\lambda_B - \lambda_F)} \quad (4.9)$$

$$g(\lambda_B, \lambda_F) = e^{i\pi s^+ (\lambda_B A - \lambda_F B)} [\lambda_B(1 + A) - \lambda_F(1 + B)]^2. \quad (4.10)$$

We introduce an indefinite integral of  $f(\lambda_B, \lambda_F)$ :  $\frac{\partial^2}{\partial \lambda_F \partial \lambda_B} F(\lambda_B, \lambda_F) = f(\lambda_B, \lambda_F)$ ,

$$\begin{aligned}
 F(\lambda_B, \lambda_F) &= - \int_{-\infty}^{\lambda_F} db \int_{\lambda_B}^{\infty} da e^{i\pi s^+(a-b)} \frac{1}{(a-b)^2} \\
 &= - \int_1^{\infty} dx e^{i\pi s^+(\lambda_B - \lambda_F)x} \left( \frac{1}{x} - \frac{1}{x^2} \right)
 \end{aligned}
 \tag{4.11}$$

where we have used the transformation  $(a-b)^{-2} = \int_0^{\infty} t e^{-t(a-b)} dt$  and  $i\pi s^+ - t = i\pi s^+ x$ . Then partial integration gives

$$\begin{aligned}
 I(s, W) &= \left[ [F(\lambda_B, \lambda_F)g(\lambda_B, \lambda_F)]_{\lambda_B=1}^{\lambda_B=\infty} \right]_{\lambda_F=-1}^{\lambda_F=1} - \left[ \int_1^{\infty} d\lambda_B F(\lambda_B, \lambda_F) \frac{\partial}{\partial \lambda_B} g(\lambda_B, \lambda_F) \right]_{\lambda_F=-1}^{\lambda_F=1} \\
 &\quad - \left[ \int_{-1}^1 d\lambda_F F(\lambda_B, \lambda_F) \frac{\partial}{\partial \lambda_F} g(\lambda_B, \lambda_F) \right]_{\lambda_B=1}^{\lambda_B=\infty} \\
 &\quad + \int_1^{\infty} d\lambda_B \int_{-1}^1 d\lambda_F F(\lambda_B, \lambda_F) \frac{\partial^2}{\partial \lambda_B \partial \lambda_F} g(\lambda_B, \lambda_F).
 \end{aligned}
 \tag{4.12}$$

After  $\lambda$ -integration, we obtain

$$\begin{aligned}
 I(s, W) &= - \left[ \left[ e^{i\pi s^+(\lambda_B A - \lambda_F B)} \int_1^{\infty} dx e^{i\pi s^+(\lambda_B - \lambda_F)x} \mathcal{I}(\lambda_B, \lambda_F, x) \right]_{\lambda_B=1}^{\lambda_B=\infty} \right]_{\lambda_F=-1}^{\lambda_F=1} \\
 &= e^{i\pi s^+(A-B)} \int_1^{\infty} dx \mathcal{I}(1, 1, x) - e^{i\pi s^+(A+B)} \int_1^{\infty} dx e^{2i\pi s^+x} \mathcal{I}(1, -1, x)
 \end{aligned}
 \tag{4.13}$$

where

$$\begin{aligned}
 \mathcal{I}(\lambda_B, \lambda_F, x) &= \left( \frac{1}{x} - \frac{1}{x^2} \right) \hat{A}^{-1} \hat{B}^{-1} [\lambda_B(1+A) - \lambda_F(1+B)]^2 \\
 &\quad - \frac{2}{i\pi s^+} \left( \frac{1}{x^2} - \frac{1}{x^3} \right) \hat{A}^{-2} \hat{B}^{-2} [\hat{A}(1+B) + \hat{B}(1+A)] [\lambda_B(1+A) - \lambda_F(1+B)] \\
 &\quad + \frac{2}{(i\pi s^+)^2} \left( \frac{1}{x^3} - \frac{1}{x^4} \right) \hat{A}^{-3} \hat{B}^{-3} [\hat{A}^2(1+B)]^2 \\
 &\quad + \hat{A} \hat{B} (1+A)(1+B) + \hat{B}^2(1+A)^2.
 \end{aligned}
 \tag{4.14}$$

Here,  $\hat{A} = 1 + \frac{A}{x}$  and  $\hat{B} = 1 + \frac{B}{x}$ . From equation (4.13) we can read that the propagator does not include zero mode variables. Therefore the difficulty of KM's approach mentioned in section 2 is resolved. We write the contribution by the first (second) term of equation (4.13) to  $R_2(s)$  as  $R_2^{(1,1)}(s)$  ( $R_2^{(1,-1)}(s)$ ).

### 5. Integration over the non-zero-momentum mode variables

In order to calculate the remaining integral over the non-zero mode variables  $W$  using the Wick theorem, we need to know the contraction rules for the action

$$S_0 = \frac{\pi}{4V} \int d\vec{r} \text{str} \left[ -\frac{D}{\Delta} (\nabla W(\vec{r}))^2 + i s^+ (\lambda_B P(B) + \lambda_F P(F)) W(\vec{r})^2 \right].
 \tag{5.1}$$

The result is summarized as

$$\langle \text{str} \mathcal{X} W(\vec{r}) \mathcal{Y} W(\vec{r}') \rangle = -\frac{1}{2} \sum_{g, g'} \Pi(\vec{r}, \vec{r}')_{gg'} [\text{str} P(g) \mathcal{X} \text{str} P(g') \mathcal{Y} - \text{str} \Lambda P(g) \mathcal{X} \text{str} \Lambda P(g') \mathcal{Y}]
 \tag{5.2}$$

$$\langle \text{str } \mathcal{X} W(\vec{r}) \text{ str } \mathcal{Y} W(\vec{r}') \rangle = -\frac{1}{2} \sum_{g, g'} \Pi(\vec{r}, \vec{r}')_{gg'} [\text{str } P(g) \mathcal{X} P(g') \mathcal{Y} - \text{str } \Lambda P(g) \mathcal{X} \Lambda P(g') \mathcal{Y}] \tag{5.3}$$

where  $\mathcal{X}$  and  $\mathcal{Y}$  are arbitrary  $4 \times 4$  supermatrices,  $g$  and  $g'$  are  $B$  or  $F$ ,

$$\langle \dots \rangle = \mathcal{D}^{-1}(s, \lambda_B, \lambda_F) \int dW(\dots) e^{-S_0} \tag{5.4}$$

$$\mathcal{D}(s, \lambda_B, \lambda_F) = \prod_{\vec{q} \neq \vec{0}} \frac{\Pi(q; \lambda_B, \lambda_B) \Pi(q; \lambda_F, \lambda_F)}{\Pi(q; \lambda_B, \lambda_F)^2} \tag{5.5}$$

$$\Pi(q; \lambda_g, \lambda_{g'}) = \frac{2V}{\pi} \left( \frac{D}{\Delta} q^2 - i s^+ \frac{\lambda_g + \lambda_{g'}}{2} \right)^{-1} \tag{5.6}$$

$$\Pi(\vec{r}, \vec{r}')_{gg'} = \sum_{\vec{q} \neq \vec{0}} \phi_{\vec{q}}(\vec{r}) \phi_{\vec{q}}^*(\vec{r}') \Pi(q; \lambda_g, \lambda_{g'}). \tag{5.7}$$

For  $R_1(s)$ , using the contraction rules with  $\lambda_B = 1$  and  $\lambda_F = 1$  we obtain

$$\begin{aligned} R_1(s) &= \frac{1}{16V^2} \text{Re} \left\langle 16V^2 + \frac{1}{4} \left( \int d\vec{r} \text{str } k W(\vec{r})^2 \right)^2 \right\rangle_{(\lambda_B, \lambda_F)=(1,1)} + O(1/g^3) \\ &= 1 + \frac{1}{8V^2} \text{Re} \sum_{\vec{q} \neq \vec{0}} \Pi(q; 1, 1)^2 + O(1/g^3) \\ &= 1 - \frac{1}{4\pi^2} \frac{d^2}{ds^2} \ln \mathcal{D}(s, 1, -1) + O(1/g^3). \end{aligned} \tag{5.8}$$

In order to get  $R_2(s)$ , we need to estimate the order of each term of equation (4.14) using the fact that (i)  $W^2 \sim O(1/g)$  for all  $s$ , (ii) the property of the exponential integral

$$E_n(a) = \int_1^\infty \frac{e^{-ax}}{x^n} dx \tag{5.9}$$

where  $\text{Re } a > 0$  and  $n \in \mathbb{N}$ , and (iii) KM's result [6]: for  $s < 1$ ,

$$R(s) = \sum_{n=0}^\infty \frac{C_n(s)}{g^n} \tag{5.10}$$

where  $C_n(s)$  is  $O(1)$ .

5.1.  $(\lambda_B, \lambda_F) = (1, 1)$  point

Expanding  $\mathcal{I}(1, 1, x)$  in  $W$ , there appears a term proportional to  $1/x, (A - B)^2/x$ . The  $x$ -integration of this term, at first glance, seems to diverge. Actually, it causes no problem because it vanishes after  $W$ -integration. In order to obtain the corrections up to  $1/g^2$ , it is sufficient to expand the remaining terms up to the fourth order of  $W$  for the entire range of  $s$ ,<sup>2</sup>

<sup>2</sup> Equation (4.14) contains  $1/s$  and  $1/s^2$  factors. One may wonder what order of  $1/g$  should be assigned to these factors because they can become arbitrarily large when  $s$  goes to 0. However KM's result (5.10) shows that these factors combined with all other  $s$  dependent terms give no divergence at  $s = 0$ . Hence we need not take these factors into account for order estimation.

$$\begin{aligned} \mathcal{I}(1, 1, x) = & -\frac{1}{x^2}(A - B)^2 - \frac{2}{i\pi s^+} \left[ 2 \left( \frac{1}{x^2} - \frac{1}{x^3} \right) (A - B) + \left( \frac{1}{x^2} - \frac{4}{x^3} + \frac{3}{x^4} \right) (A^2 - B^2) \right] \\ & + \frac{2}{(i\pi s^+)^2} \left[ 3 \left( \frac{1}{x^3} - \frac{1}{x^4} \right) + 3 \left( \frac{1}{x^3} - \frac{3}{x^4} + \frac{2}{x^5} \right) (A + B) \right. \\ & \left. + \left( \frac{1}{x^3} - \frac{9}{x^4} + \frac{18}{x^5} - \frac{10}{x^6} \right) (A^2 + B^2 + AB) \right] + O(1/g^3). \end{aligned} \tag{5.11}$$

After  $x$ -integration, we obtain

$$\int_1^\infty \mathcal{I}(1, 1, x) dx = -(A - B)^2 - \frac{2}{i\pi s^+}(A - B) + \frac{1}{(i\pi s^+)^2} + O(1/g^3). \tag{5.12}$$

After  $W$ -integration, we obtain

$$R_2^{(1,1)}(s) = -\frac{1}{2\pi^2 s^2}. \tag{5.13}$$

5.2.  $(\lambda_B, \lambda_F) = (1, -1)$  point

For  $s \geq 1$ ,  $E_n(-i2\pi s^+)$  in equation (4.13) is  $O(1)$ . Accordingly, for the same reason as the case of the  $(\lambda_B, \lambda_F) = (1, 1)$  point, it is again sufficient to expand  $\mathcal{I}(1, -1, x)$  up to the fourth order of  $W$  for the entire range of  $s$ .

$$\begin{aligned} \mathcal{I}(1, -1, x) = & 4 \left[ \left( \frac{1}{x} - \frac{1}{x^2} \right) + \frac{4}{\tilde{s}} \left( \frac{1}{x^2} - \frac{1}{x^3} \right) + \frac{6}{\tilde{s}^2} \left( \frac{1}{x^3} - \frac{1}{x^4} \right) \right] \\ & + 4 \left[ \left( \frac{1}{x} - \frac{2}{x^2} + \frac{1}{x^3} \right) + \frac{2}{\tilde{s}} \left( \frac{2}{x^2} - \frac{5}{x^3} + \frac{3}{x^4} \right) + \frac{6}{\tilde{s}^2} \left( \frac{1}{x^3} - \frac{3}{x^4} + \frac{2}{x^5} \right) \right] (A + B) \\ & + \left[ \left( \frac{1}{x} - \frac{5}{x^2} + \frac{7}{x^3} - \frac{3}{x^4} \right) + \frac{4}{\tilde{s}} \left( \frac{1}{x^2} - \frac{7}{x^3} + \frac{12}{x^4} - \frac{6}{x^5} \right) \right. \\ & \left. + \frac{6}{\tilde{s}^2} \left( \frac{1}{x^3} - \frac{9}{x^4} + \frac{18}{x^5} - \frac{10}{x^6} \right) \right] (A + B)^2 + \left[ \left( \frac{1}{x^3} - \frac{1}{x^4} \right) \right. \\ & \left. - \frac{4}{\tilde{s}} \left( \frac{1}{x^3} - \frac{3}{x^4} + \frac{2}{x^5} \right) + \frac{2}{\tilde{s}^2} \left( \frac{1}{x^3} - \frac{9}{x^4} + \frac{18}{x^5} - \frac{10}{x^6} \right) \right] (A - B)^2 + O(1/g^3) \end{aligned} \tag{5.14}$$

where  $\tilde{s} = -i2\pi s^+$ . Using the relation  $E_n(\tilde{s}) + \frac{n}{\tilde{s}} E_{n+1}(\tilde{s}) = e^{-\tilde{s}}/\tilde{s}$ , we obtain

$$\int_1^\infty dx e^{-\tilde{s}x} \mathcal{I}(1, -1, x) = 4 \frac{e^{-\tilde{s}}}{\tilde{s}^2} + \frac{1}{\tilde{s}^2} \left[ e^{-\tilde{s}} + 2 \int_1^\infty dx e^{-\tilde{s}x} \left( \frac{1}{x^3} - \frac{3}{x^4} \right) \right] (A - B)^2 + O(1/g^3) \tag{5.15}$$

$$\begin{aligned} e^{i\pi s^+(A+B)} \int_1^\infty dx e^{i2\pi s^+x} \mathcal{I}(1, -1, x) = & e^{i\pi s^+(A^{(2)}+B^{(2)})} \frac{e^{i2\pi s^+}}{(i\pi s^+)^2} \\ & + e^{i\pi s^+(A^{(2)}+B^{(2)})} \left[ \frac{e^{i2\pi s^+}}{i\pi s^+} (A^{(4)} + B^{(4)}) + \frac{\mathcal{F}(s)}{4(i\pi s^+)^2} (A^{(2)} - B^{(2)})^2 \right] + O(1/g^3) \end{aligned} \tag{5.16}$$

where  $\mathcal{F}(s) = e^{i2\pi s^+} + 2 \int_1^\infty dx e^{i2\pi s^+x} \left( \frac{1}{x^3} - \frac{3}{x^4} \right)$ . The contribution by the second term of equation (5.16) is  $O(1/g^3)$ , because for  $s \geq g$  it is obvious and for  $s < g$  it is found after  $W$ -integration.

In the same way, the contributions by  $O(W^4)$  terms emerging from  $(\nabla \tilde{Q}(\vec{r}))^2$  and the Jacobian  $J(W)$  turn out to be  $O(1/g^3)$ . Finally, we obtain

$$R_2^{(1,-1)}(s) = -\frac{1}{2} \operatorname{Re} \int dW J(W) \exp \left[ -\frac{\pi D}{4V\Delta} \int d\vec{r} \operatorname{str}(\nabla \tilde{Q}(\vec{r}))^2 \right] e^{i\pi s^+(A^{(2)}+B^{(2)})} \frac{e^{i2\pi s^+}}{(i\pi s^+)^2} + O(1/g^3) = \frac{\cos 2\pi s}{2\pi^2 s^2} \mathcal{D}(s, 1, -1) + O(1/g^3). \tag{5.17}$$

### 6. Result and summary

Equations (5.8), (5.13) and (5.17) give

$$R(s) = 1 - \frac{1}{4\pi^2} \frac{d^2}{ds^2} \ln \mathcal{D}(s, 1, -1) - \frac{1}{2\pi^2 s^2} + \frac{\cos 2\pi s}{2\pi^2 s^2} \mathcal{D}(s, 1, -1) + O(1/g^3) = 1 - \frac{1}{4\pi^2} \frac{d^2}{ds^2} \ln \tilde{\mathcal{D}}(s) + \frac{\cos 2\pi s}{2\pi^2} \tilde{\mathcal{D}}(s) + O(1/g^3) \tag{6.1}$$

where  $\tilde{\mathcal{D}}(s) = \mathcal{D}(s, 1, -1)/s^2$  is the spectral determinant of the classical diffusion operator, in agreement with [9–11]. Here the contribution of the  $(\lambda_B, \lambda_F) = (1, 1)$

$$-\frac{1}{4\pi^2} \frac{d^2}{ds^2} \ln \tilde{\mathcal{D}}(s) = \frac{1}{8V^2} \operatorname{Re} \sum_{\vec{q}} \Pi(\vec{q}, 1, 1)^2 \tag{6.2}$$

is the ordinary perturbative result [5].

In summary, using KM’s separate treatment of the zero and non-zero modes, performing the integration over the zero modes exactly and subsequent integration over the non-zero modes perturbatively, we find the expression for the two-point energy level correlation function valid for arbitrary energy differences up to  $1/g^2$  order for unitary ensemble. This expression is same as that [9–11] obtained in a saddle-point approximation which in general is valid only for  $\omega \gg \Delta$ .<sup>3</sup> By explicit calculation, we verify that the exactness of the saddle-point answer for the unitary ensemble, which is guaranteed in the ergodic regime due to the Duistermaat–Heckman theorem [15], holds even in the diffusive regime. Since this is a specific feature of the unitary ensemble, it is very interesting to investigate the expressions for the others (orthogonal, symplectic, etc).

The reason why AA’s novel saddle point appears is as follows. The zero mode integral is carried out exactly. Then the remaining expression for non-zero modes is evaluated at terminal points of the integral region,  $1 \leq \lambda_B < \infty, -1 \leq \lambda_F < 1$ . These terminal points precisely correspond to the ordinary  $(\lambda_B = \lambda_F = 1)$  and AA’s novel saddle point  $(\lambda_B = 1, \lambda_F = -1)$ . This correspondence between the terminal points of zero mode variables and saddle points seems to hold for the orthogonal ensemble. Hence, for the orthogonal case, a similar calculation may work: although exact integration of zero mode variables is no longer probable, the iteration of partial integration enables one to evaluate the integral on the terminal points up to a necessary order of  $1/g$ , if the additional terms generated by these iterations are only higher order terms of  $1/g$ . For the symplectic case, the correspondence seems more subtle because the novel saddle points are not isolated but

<sup>3</sup> In AA’s calculation, technically speaking, they had to introduce an external parameter in order to avoid the divergence of their integral over zero mode variables of the fermion–fermion block for the novel saddle point. However, such an artificial regulator is not necessary in our calculation.

consist of a saddle-point manifold. Further investigation will be necessary for the symplectic case.

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### Appendix A. Calculation of the Jacobian

We give the calculation of the Jacobian  $J$  of the transformation  $Q \rightarrow \{W, T_0\}$  because, to the best of our knowledge, there is no explicit expression of it in the literature. It is calculated by the following Gaussian integral:

$$\int d(\delta W) d(\delta T_0) \exp \left[ - \int d\vec{r} \operatorname{str}(\delta Q(\vec{r}))^2 \right] = J^{-1}. \quad (\text{A1})$$

Here  $\delta Q(\vec{r})$  is a variation of equation (2.3),

$$\delta Q(\vec{r}) = T_0^{-1}(\delta \tilde{Q}(\vec{r}) + [\tilde{Q}(\vec{r}), \delta T_0'])T_0 \quad (\text{A2})$$

where  $\delta T_0' = \delta T_0 T_0^{-1}$ . Then,

$$\int d\vec{r} \operatorname{str}(\delta Q(\vec{r}))^2 = \int d\vec{r} \operatorname{str}(\delta \tilde{Q}(\vec{r}))^2 + 2 \int d\vec{r} \operatorname{str} \delta \tilde{Q}(\vec{r})[\tilde{Q}(\vec{r}), \delta T_0'] + \int d\vec{r} \operatorname{str}[\tilde{Q}(\vec{r}), \delta T_0']^2. \quad (\text{A3})$$

The integrand of equation (A1) is expanded up to fourth order in  $W$ .

The  $\delta W$  dependent part of the integrand is

$$\begin{aligned} \exp \left[ - \int d\vec{r} \operatorname{str}(\delta \tilde{Q}(\vec{r}))^2 - 2 \int d\vec{r} \operatorname{str} \delta \tilde{Q}(\vec{r})[\tilde{Q}(\vec{r}), \delta T_0'] \right] &= \exp \left[ \int d\vec{r} \operatorname{str}(\delta W)^2 \right] \\ &\times \left\{ 1 + \frac{1}{2} \int d\vec{r} \operatorname{str} W^2(\delta W)^2 + 2 \left( \int d\vec{r} \operatorname{str} X_1(\delta W)\delta T_0' \right)^2 \right. \\ &+ 2 \left( \int d\vec{r} \operatorname{str} X_1(\delta W)\delta T_0' \right) \left( \int d\vec{r} \operatorname{str} X_2(\delta W)\delta T_0' \right) \\ &+ \frac{1}{8} \int d\vec{r} \operatorname{str} W^4(\delta W)^2 + \frac{1}{16} \int d\vec{r} \operatorname{str}(W^2\delta W)^2 + \frac{1}{8} \left[ \int d\vec{r} \operatorname{str} W^2(\delta W)^2 \right]^2 \\ &+ \left( \int d\vec{r} \operatorname{str} X_1(\delta W)\delta T_0' \right) \left( \int d\vec{r} \operatorname{str} X_3(\delta W)\delta T_0' \right) \\ &+ \frac{1}{2} \left( \int d\vec{r} \operatorname{str} X_2(\delta W)\delta T_0' \right)^2 + \frac{2}{3} \left( \int d\vec{r} \operatorname{str} X_1(\delta W)\delta T_0' \right)^4 \\ &\left. + \left[ \int d\vec{r} \operatorname{str} W^2(\delta W)^2 \right] \left( \int d\vec{r} \operatorname{str} X_1(\delta W)\delta T_0' \right)^2 + O(W^5) \right\} \quad (\text{A4}) \end{aligned}$$

where

$$X_n(\delta W) = \sum_{k=0}^n (-1)^k W^k \delta W W^{n-k}. \quad (\text{A5})$$

The  $\delta W$  independent part is

$$\begin{aligned} \exp\left(-\int d\vec{r} \operatorname{str}[\tilde{Q}(\vec{r}), \delta T_0']^2\right) &= \exp[2V \operatorname{str}(\delta T_0' - \Lambda \delta T_0' \Lambda) \delta T_0'] \\ &\times \left\{ 1 - 2 \int d\vec{r} \operatorname{str} \Lambda \delta T_0' \Lambda (W^2 \delta T_0' - W \delta T_0' W) \right. \\ &- \int d\vec{r} \operatorname{str} \Lambda \delta T_0' \Lambda (W^3 \delta T_0' - 2W^2 \delta T_0' W) \\ &- \frac{1}{2} \int d\vec{r} \operatorname{str} \Lambda \delta T_0' \Lambda (W^4 \delta T_0' - 2W^3 \delta T_0' W + W^2 \delta T_0' W^2) \\ &\left. + 2 \left[ \int d\vec{r} \operatorname{str} \Lambda \delta T_0' \Lambda (W^2 \delta T_0' - W \delta T_0' W) \right]^2 + O(W^5) \right\}. \end{aligned} \quad (\text{A6})$$

The integral of the  $\delta W$  dependent part over  $\delta W$  can be calculated by using the Wick theorem and the contraction rules,

$$\langle \operatorname{str} \mathcal{X} \delta W(\vec{r}) \mathcal{Y} \delta W(\vec{r}') \rangle_{\delta W} = \frac{1}{4} \left( \frac{1}{V} - \delta(\vec{r} - \vec{r}') \right) (\operatorname{str} \mathcal{X} \operatorname{str} \mathcal{Y} - \operatorname{str} \Lambda \mathcal{X} \operatorname{str} \Lambda \mathcal{Y}) \quad (\text{A7})$$

$$\langle \operatorname{str} \mathcal{X} \delta W(\vec{r}) \operatorname{str} \mathcal{Y} \delta W(\vec{r}') \rangle_{\delta W} = \frac{1}{4} \left( \frac{1}{V} - \delta(\vec{r} - \vec{r}') \right) (\operatorname{str} \mathcal{X} \mathcal{Y} - \operatorname{str} \Lambda \mathcal{X} \Lambda \mathcal{Y}) \quad (\text{A8})$$

where  $\mathcal{X}$  and  $\mathcal{Y}$  are arbitrary supermatrices and

$$\langle \dots \rangle_{\delta W} = \int d(\delta W) (\dots) \exp \left[ \int d\vec{r} \operatorname{str}(\delta W)^2 \right]. \quad (\text{A9})$$

The result is

$$\begin{aligned} &\int d(\delta W) \exp \left[ - \int d\vec{r} \operatorname{str}(\delta \tilde{Q}(\vec{r}))^2 - 2 \int d\vec{r} \operatorname{str} \delta \tilde{Q}(\vec{r}) [\tilde{Q}(\vec{r}), \delta T_0'] \right] \\ &= 1 + \int d\vec{r} \operatorname{str}(\delta T_0' + \Lambda \delta T_0' \Lambda) (W^2 \delta T_0' - W \delta T_0' W) \\ &+ \int d\vec{r} \operatorname{str} \Lambda \delta T_0' \Lambda (W^3 \delta T_0' - 2W^2 \delta T_0' W) - \frac{1}{8} \int d\vec{r} \operatorname{str}(W^2 \delta T_0')^2 \\ &+ \frac{1}{8} \int d\vec{r} \operatorname{str} \Lambda \delta T_0' \Lambda (4W^4 \delta T_0' - 8W^3 \delta T_0' W + 5W^2 \delta T_0' W^2) \\ &+ \frac{1}{2} \left[ \int d\vec{r} \operatorname{str}(\delta T_0' + \Lambda \delta T_0' \Lambda) (W^2 \delta T_0' - W \delta T_0' W) \right]^2 \\ &+ \frac{1}{8V} \int d\vec{r} d\vec{r}' \operatorname{str}(\delta T_0' - \Lambda \delta T_0' \Lambda) (2W(\vec{r})^2 W(\vec{r}')^2 \delta T_0' - 2W(\vec{r})^2 W(\vec{r}') \delta T_0' W(\vec{r}')) \\ &- 2W(\vec{r}) W(\vec{r}')^2 \delta T_0' W(\vec{r}') + 2W(\vec{r})^2 \delta T_0' W(\vec{r}')^2 + W(\vec{r}) W(\vec{r}') \delta T_0' W(\vec{r}') W(\vec{r})) \\ &- \frac{1}{64V} \int d\vec{r} (\operatorname{str} W^2)^2 + \frac{1}{64V^2} \left( \int d\vec{r} \operatorname{str} W^2 \right)^2 + O(W^5). \end{aligned} \quad (\text{A10})$$

Multiplying this by equation (A6), we find

$$\begin{aligned} &\int d(\delta W) \exp \left[ - \int d\vec{r} \operatorname{str}(\delta \tilde{Q}(\vec{r}))^2 \right] = \exp[2V \operatorname{str}(\delta T_0' - \Lambda \delta T_0' \Lambda) \delta T_0'] \\ &\times \left\{ 1 + \int d\vec{r} \operatorname{str}(\delta T_0' - \Lambda \delta T_0' \Lambda) (W^2 \delta T_0' - W \delta T_0' W) \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \left[ \int d\vec{r} \operatorname{str}(\delta T'_0 - \Lambda \delta T'_0 \Lambda) (W^2 \delta T'_0 - W \delta T'_0 W) \right]^2 + \frac{1}{64V^2} \left( \int d\vec{r} \operatorname{str} W^2 \right)^2 \\
& + \frac{1}{8V} \int d\vec{r} d\vec{r}' \operatorname{str}(\delta T'_0 - \Lambda \delta T'_0 \Lambda) (2W(\vec{r})^2 W(\vec{r}')^2 \delta T'_0 \\
& - 2W(\vec{r})^2 W(\vec{r}') \delta T'_0 W(\vec{r}') - 2W(\vec{r}) W(\vec{r}')^2 \delta T'_0 W(\vec{r}') + 2W(\vec{r}) \delta T'_0 W(\vec{r}')^2 \\
& + W(\vec{r}) W(\vec{r}') \delta T'_0 W(\vec{r}') W(\vec{r})) - \frac{1}{8} \int d\vec{r} \operatorname{str}(\delta T'_0 - \Lambda \delta T'_0 \Lambda) W^2 \delta T'_0 W^2 \\
& - \frac{1}{64V} \int d\vec{r} (\operatorname{str} W^2)^2 + O(W^5) \Big\}. \tag{A11}
\end{aligned}$$

The remaining  $\delta T_0$  integral can also be calculated using the Wick theorem and the contraction rules,

$$\langle \operatorname{str} \mathcal{X} (\delta T'_0)_{12} \mathcal{Y} (\delta T'_0)_{21} \rangle_{\delta T_0} = -\frac{1}{8V} \operatorname{str} \mathcal{X} \operatorname{str} \mathcal{Y} \tag{A12}$$

$$\langle \operatorname{str} \mathcal{X} (\delta T'_0)_{12} \operatorname{str} \mathcal{Y} (\delta T'_0)_{21} \rangle_{\delta T_0} = -\frac{1}{8V} \operatorname{str} \mathcal{X} \mathcal{Y} \tag{A13}$$

where  $\mathcal{X}$  and  $\mathcal{Y}$  are arbitrary supermatrices and

$$\langle \dots \rangle_{\delta T_0} = \int d(\delta T_0) (\dots) \exp[2V \operatorname{str}(\delta T'_0 - \Lambda \delta T'_0 \Lambda) \delta T'_0]. \tag{A14}$$

Since the zero mode variables enter into equation (A11) only through  $(\delta T'_0)_{12}$  and  $(\delta T'_0)_{21}$ , the Jacobian does not contain the zero mode variables. We finally obtain the Jacobian (2.6) from the expression

$$\begin{aligned}
& \int d(\delta W) d(\delta T_0) \exp \left[ - \int d\vec{r} \operatorname{str}(\delta \mathcal{Q}(\vec{r}))^2 \right] \\
& = 1 + \frac{1}{16V^2} \int d\vec{r} d\vec{r}' (\operatorname{str} W_{12}(\vec{r}) W_{21}(\vec{r}'))^2 + O(W^5). \tag{A15}
\end{aligned}$$

## Appendix B. Block diagonalization

We can diagonalize  $Y = \int d\vec{r} \tilde{\mathcal{Q}}(\vec{r})$  in the retarded–advanced space as follows. For equation (3.4),

$$Y = \sum_{n=0} Y^{(n)} \tag{B1}$$

$$X = \sum_{n=0} X^{(2n+1)} \quad \{X^{(2n+1)}, \Lambda\} = 0 \tag{B2}$$

$$\hat{\mathcal{Q}} = \sum_{n=0} \hat{\mathcal{Q}}^{(2n)} \quad [\hat{\mathcal{Q}}^{(2n)}, \Lambda] = 0 \tag{B3}$$

where  $Y^{(n)}$ ,  $X^{(n)}$ ,  $\hat{\mathcal{Q}}^{(n)}$  mean  $O(W^n)$  term for  $Y$ ,  $X$ ,  $\hat{\mathcal{Q}}$  and  $\{, \}$  stands for an anticommutator. Then  $\hat{\mathcal{Q}}$  and  $X$  are perturbatively determined and given by equation (3.5) and

$$X = \frac{1}{8V} \int d\vec{r} W^3(\vec{r}) + O(W^5). \tag{B4}$$

We can parametrize  $e^X T_0$  in terms of  $\mathcal{T}$  and  $\mathcal{R}$ ,

$$e^X T_0 = \mathcal{T} \mathcal{R} \tag{B5}$$

where  $T$  and  $R$  obey the properties

$$T^{-1} = \Lambda T \Lambda \quad [R, \Lambda] = 0 \quad (\text{B6})$$

and are given by

$$T = (e^X T_0^2 e^X)^{1/2} = T_0 + O(W^3) + O(W^5) \quad (\text{B7})$$

$$R = T^{-1} e^X (e^{-X} T^2 e^{-X})^{1/2} = \mathbf{1} + O(W^3) + O(W^5). \quad (\text{B8})$$

From equation (B7),

$$dT_0 = (1 + O(W^3) + O(W^5)) dT. \quad (\text{B9})$$

Substituting the transformation (3.4) and (B5) in equations (3.2) and (3.3), we find

$$\int d\vec{r} \operatorname{str} \Lambda Q(\vec{r}) = \operatorname{str} \Lambda T^{-1} \hat{Q}T \quad (\text{B10})$$

$$\left( \int d\vec{r} \operatorname{str} k \Lambda Q(\vec{r}) \right)^2 = (\operatorname{str} k \Lambda T^{-1} \hat{Q}T)^2 + O(W^3) + O(W^5). \quad (\text{B11})$$

In equations (B9) and (B11),  $O(W^3)$  terms contribute to  $O(W^6)$  in  $R(s)$  and are negligible. Therefore we replace  $T$  with  $T_0$ .

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